Chapter 2

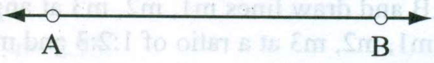
Review of Geometry

2.1 Geometry

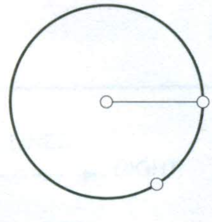
The word geometry is derived from two Greek words meaning earth measurement. It seems probable that many of the early discoveries in geometry were motivated by the need to make measurements of distances and areas on the Earth. However, Euclidean geometry has a broader meaning. It is the chief subject matter of the monumental 13-volume work called *'The Elements,'* written about 300 B.C. by the Greek mathematician Euclid, who taught and founded a school of geometry at Alexandria. One of the milestones in the history of scientific thought, these books of Euclid still occupies an important position in mathematical instruction today[6].

Now look at some basic Ideas of Geometry - Points, Lines, Planes and Angles[7]

- **point** - marks a location, has no thickness, no width, and no height
- **line** - through any two points, there is exactly one line, unlimited length, straight, no end points no thickness
- **plane** - Three non-collinear points determine a plane, no boundary, no thickness, flat, does not curve, continues in all directions
- **collinear** - points on same line
- **coplanar** - points that all lie in one plane
- **intersecting lines** - Two lines with one point in common
- **parallel lines** - Two lines in the same plane that do not intersect
- **concurrent lines** - Three or more coplanar lines that have a point in common
- **skew lines** - Two non-coplanar lines that do not intersect
- **parallel planes** - planes that have no points in common
- **intersecting planes** - planes that have a line in common
- **segment** - AB set of all points A,B and all points between A and B



- **circle** - set of all points equidistant from a center point
- **acute** - less than a 90 degree angle
- **right** - 90 degrees
- **obtuse** - greater than 90 degrees
- **Oblique Angle**[8] - An angle that is not a right angle.



2.2 Two Dimensional Geometric Constructions

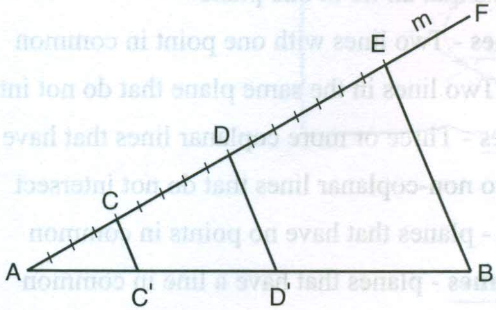
Do you remember triangles, rectangles, squares and other geometric figures? All of them are two- dimensional (2-D). To define a point on 2-D, you need 'X' and 'Y' coordinates (i.e. two measurements). It is also called plane geometry, since they all lie in one plane.

There are different ways to draw common geometric figures. Here, the following methods[9] are not hard as examples. However, some of these graphical solutions are common and some are not. As discussed earlier, Greek mathematician Euclid around 300 BC (Over 2300 years ago!) wrote thirteen books on geometry and in Book IV, you can find how to draw a regular hexagon with straightedge and compass.[9]

2.2.A. The Straight Line and Its Division

1. *To Divide a Line Segment into a Given Ratio*

Let AB represent the given line segment, and let the given ratio be 4:5:7. Draw line "m" (i.e. line AF) through point A and at any convenient angle with AB. On "m" lay off distances AC = 4 units, CD = 5 units, DE = 7 units. Draw BE and then draw lines through C and D parallel to BE, cutting AB in points C' and D', which determine the required segments of AB.



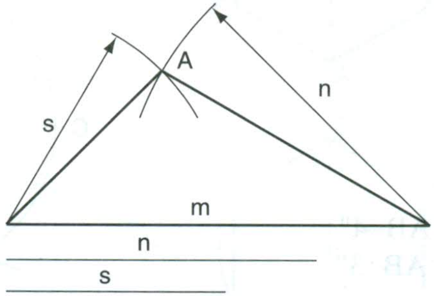
Exercise :

Take any line length AB and draw lines m1, m2, m3 at angles 30, 60 and 90 degrees to AB, then cut the lines m1, m2, m3 at a ratio of 1:2:3 and mark the points as C1, D1, E1, and C2, D2, E2, and C3, D3, E3. Then continue as above.

2.2.B. The Construction of Triangles and Regular Polygons

1. ***To Construct a Triangle with Known Lengths of the Sides***

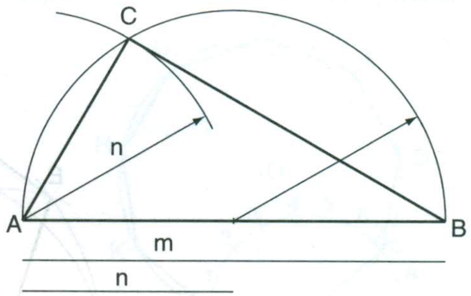
Suppose m, n, and s are the given lengths. With the end points of m as centers and radii equal to n and s, respectively, draw intersecting arcs, locating point A. Join A with the end points of m to complete the construction of the triangle. Is it possible to construct a triangle with 7, 3.5 and 3 inches as the lengths?



Exercise : Take m= 5″, n= 4″, s =3″
Take m= 4″, n= 3″, s =2″

2. ***To Construct a Right Triangle when the Lengths of the Hypotenuse and One Side are Known***

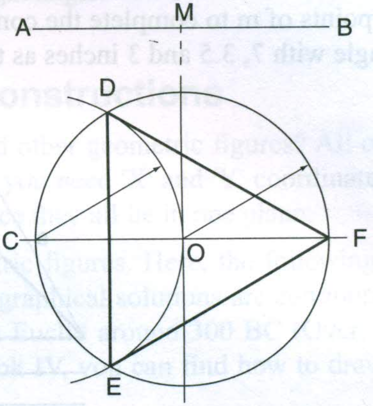
Let line m and n represent the given lengths. Construct a semicircle with diameter AB equal to length m. With A as center (could use B) and radius equal to length n, draw an arc cutting the semicircle at point C. Triangle ABC is the required right triangle. Why is this true?



Exercise : Take m= 6″, n= 2.75″
Take m= 4″, n= 1.5″

3. *To Inscribe an Equilateral Triangle within a Circle Having a Given Diameter, AB*

With center O and radius equal to AM (M is midpoint of AB), draw the circle. With C as center and the same length of radius, draw an arc cutting the circle in points D and E. Join points D, E, and F. The required triangle is DEF.



Exercise : Take AB=4"
Take AB=3"

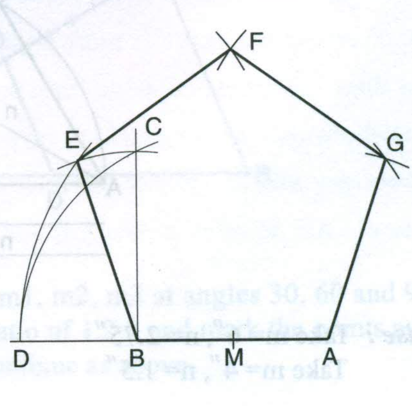
4. *To Construct a Regular Pentagon when the Length AB of the Sides is known*

i) First construct BC = AB, and BC is perpendicular to AB.

ii) With M (midpoint of AB) as center and MC as radius, draw an arc cutting AB extended at point D.

iii) Now with A as center and AD as radius, draw an arc; and, with B as center and radius BA, draw an intersecting arc to locate point E. Line BE is a side of the regular pentagon.

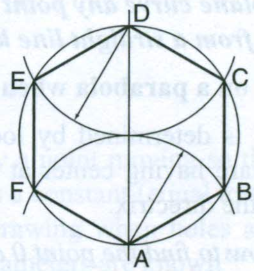
iv) Then with B as center and AD as radius, draw an arc; and, with A as center and radius BA, draw an intersecting arc to locate point G. The construction for locating point F is fairly obvious. ABEFG is the required pentagon.



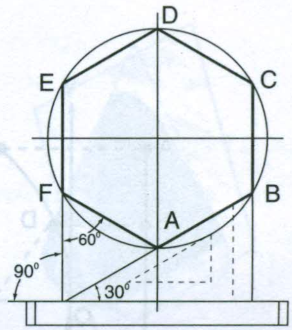
Exercise : Take AB = 2″
Take AB = 1.5″

5. *To Inscribe a Hexagon within a Given Circle*

With A and D as centers and a radius equal to the radius of the circle, draw arcs which intersect the given circle in points B, F, C, and E. The required hexagon is ABCDEF.

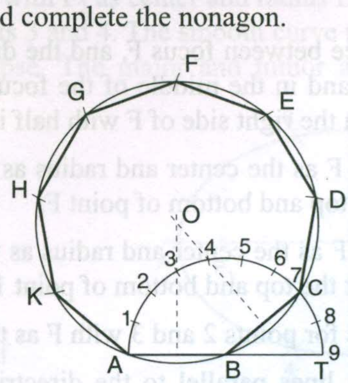


An alternative construction is shown.



6. *To Construct a Regular Polygon Having n Sides*

The polygon in this example is a nonagon (nine equal sides) and AB is the given length of each side. With B as center and AB as radius, describe a semicircle, and by trial divide it (i.e. the semicircle) into nine equal parts. Starting from point T, locate the second division mark, C. Locate point 0, the center of the circumscribing circle. (Finding the intersection of the perpendicular bisectors of AB and BC easily does Point 0.) Draw the circle with center O and radius OA and complete the nonagon.



General Rule: *To find the summation of angles in any polygon, use* **(2N-4) × 90°** *where, N is the number of sides.*

2.2.C. Conic Sections

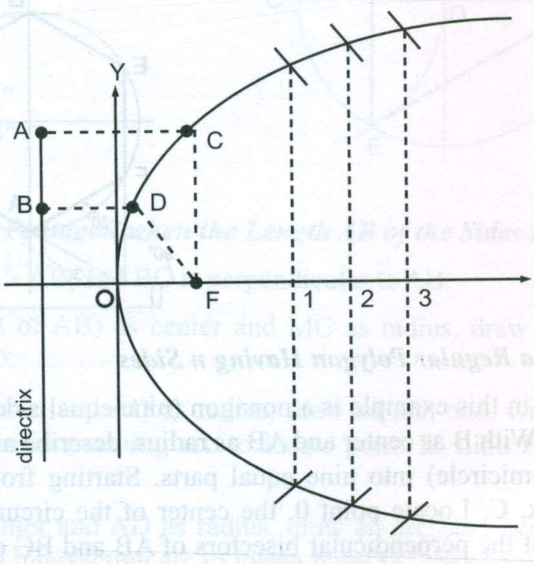
Parabola

A parabola is a plane curve any point of which is the same distance from a point called the focus as it is from a straight line known as the directrix[10].

To locate points on a parabola when the focus, F, and the directrix, AB are given

Point such as C is determined by locating the intersection of any line parallel to the directrix and an arc having center at F and a radius equal to the distance between the parallel line and the directrix.

Could you tell how to find the point 0 on the following figure?



Exercise :

Take the distance between focus F, and the directrix, AB as two inches. Therefore point O is on X-axis and in the middle of the focus F, and the directrix, AB. Now take three points (1,2,3) on the right side of F with half inches interval.

1. Then, take F as the center and radius as the distance between F and AB, draw arcs on just the top and bottom of point F.
2. Then, take F as the center and radius as the distance between point 1 and AB, draw arcs on just the top and bottom of point 1.
3. Repeat this for points 2 and 3 with F as the center.
4. Now draw lines parallel to the directrix through points F, 1,2 and 3 to find the intersections on the top and bottom of the X-axis. Join all these points and pass through point O to complete the parabola.

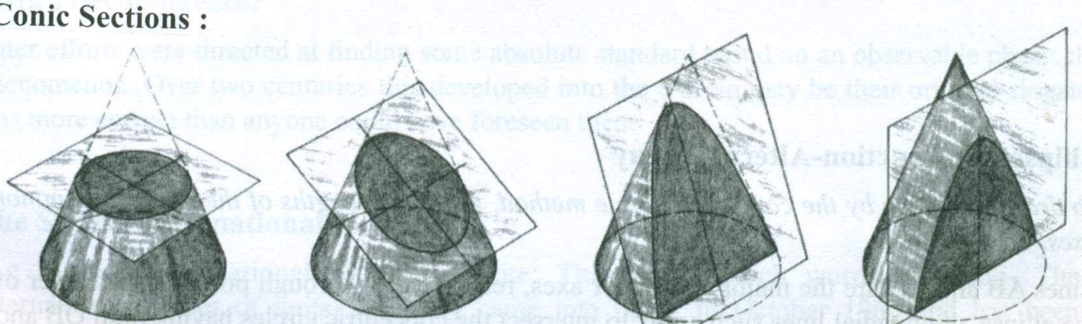
Ellipse:

The ellipse may be defined as the locus of all coplanar points, the sum of whose distances from two fixed points (foci) is a constant.

More notes on ellipse.

Mathematically the ellipse is a curve generated by a point moving so that at any position the sum of its distances from two fixed points (foci) is a constant (equal to the major diameter). It is encountered very frequently in orthographic drawing when holes and circular forms are view obliquely. Ordinarily, the major and minor diameters are known[11]

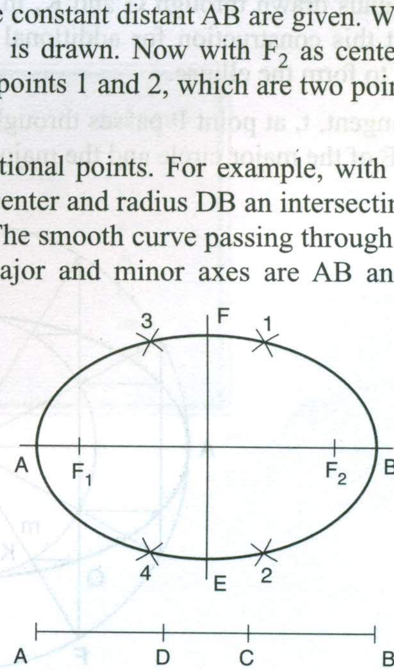
Conic Sections :



Circle Ellipse Parabola Hyperbola

To construct an ellipse when the foci F_1 and F_2 and the constant distant AB are given. With F_1 as center and radius AC (any portion of AB), an arc is drawn. Now with F_2 as center and radius CB, an arc is drawn intersecting the first arc in points 1 and 2, which are two points on the ellipse.

This construction is repeated for the location of additional points. For example, with F_1 as center and radius AD, an arc is drawn, and with F_2 as center and radius DB an intersecting arc is drawn, thus locating two additional points 3 and 4. The smooth curve passing through these points and others (not shown) is the ellipse. The major and minor axes are AB and EF, respectively.



Exercise :

Take AB as 4 inches and F_1, F_2 half inches from A,B respectively. Also, take AD=1.5" and BC=1.5". Find the points 1,2,3,4 as showed above.

Exercise :

Take

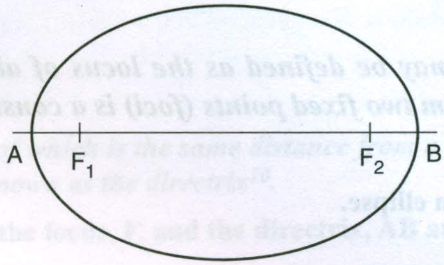
AB = 4″

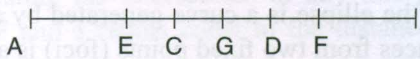
$AF_1 = BF_2 = 0.5''$

AC = BD = 1.5″

AE = BF = 1″

AG = BG = 2″



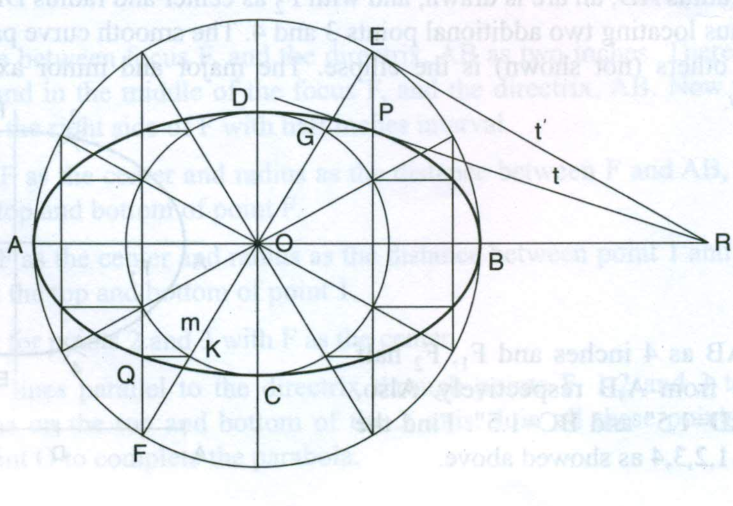


Ellipse Construction-Alternate Way

To draw an ellipse by the concentric circle method, given the lengths of the major and minor axes.

Lines AB and CD are the major and minor axes, respectively. Through point O, the center of the ellipse, draw radial lines such as 'm' to intersect the concentric circles having radii OB and OC in points E, F, G, and K. Through points E and F draw vertical lines to intersect the horizontals drawn through G and K, in points P and Q, which are two points on the ellipse. Repeat this construction for additional points and then draw a smooth curve through these points to form the ellipse.

The tangent, t, at point P passes through point R, which is the intersection of the tangent t' at point E of the major circle and the major axis extended.



2.3 A Brief History of Measurement

One of the earliest types of measurement concerned that of length. These measurements were usually based on parts of the body. A well documented example (the first) is the Egyptian cubit which was derived from the length of the arm from the elbow to the outstretched finger tips. By 2500 BC (4500 years from today!) this had been standardized in a royal master cubit made of black marble (about 52 cm). This cubit was divided into 28 digits (roughly a finger width) which could be further divided into fractional parts, the smallest of these being only just over a millimeter.

In France the metric system officially started in June 1799 with the declared intent of being 'For all people, for all time'. **The unit of length was the meter which was defined as being one ten-millionth part of a quarter of the earth's circumference. Could you calculate the earth's circumference?**

Later efforts were directed at finding some absolute standard based on an observable physical phenomenon. Over two centuries this developed into the **S I**. So may be their original slogan was more correct than anyone could have foreseen then.

The System International [S I]

"Le Systeme international d'Unites" [Note: These are French words, meaning- The international system of units] officially came into being in October 1960 and has been officially recognized and adopted by nearly all countries, though the amount of actual usage varies considerably. It is based upon 7 principal units, 1 in each of 7 different categories -

Category	*Name*	*Abbrev.*
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	Kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

The U K (Imperial) System of Measurements

The old Imperial (now UK) system was originally defined by three standard measures - the yard, the pound and the gallon which were held in London. They are now defined by reference to the S I measures of the meter, the kilogram and the liter. These equivalent measures are exact.

1 yard	= 0.9144 meters	- same in US
1 pound	= 0.453 592 37 kilograms	- same in US
1 gallon	= 4.546 09 liters	- different in US

Notice particularly that the UK gallon is a different size to the US gallon so that NO liquid measures of the same name are the same size in the UK and US systems. Also that **the ton (UK) is 2240 pounds while a ton (US) is 2000 pounds.** These are also referred to as a **long ton** and **short ton** respectively[12].

2.4 Coordinate System

A system for specifying points using coordinates measured in some specified way. The simplest coordinate system consists of coordinate axes oriented perpendicularly to each other, known as *Cartesian coordinates*. Depending on the type of problem under consideration, coordinate systems possessing special properties may allow particularly simple solution. In three dimensions, so-called right-handed coordinate systems are usually chosen by convention, although left-handed coordinate systems are also encountered occasionally [13]

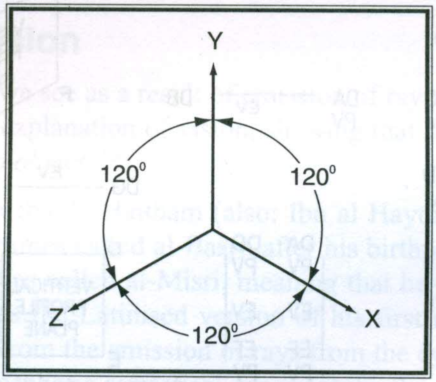
3-D Coordinate System is same as for 2-D, with the addition of a z-axis. ***For each orientation (Right-handed or Left-handed), the thumb points in the +x direction, the pointer finger is +y, and the middle finger is +z.)***[14]

3-D visualization

3-D Visualization skills are important in a variety of careers: engineering, architecture, chemistry, mathematics, sheet metal working, etc.[15]

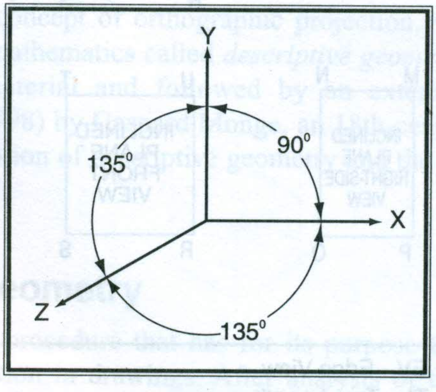
3-D Coordinate Systems Used for Visualization

Isometric Axes: Isometric means "equal measurement". In the Isometric system, each pair of axes meet at 120^0. Thus, the name isometric from the Greek iso- meaning the same. When objects are drawn using isometric axes, all surfaces of the object will appear distorted. In other words, squares will appear as parallelograms, and circles will appear as ellipses.



Isometric Axes

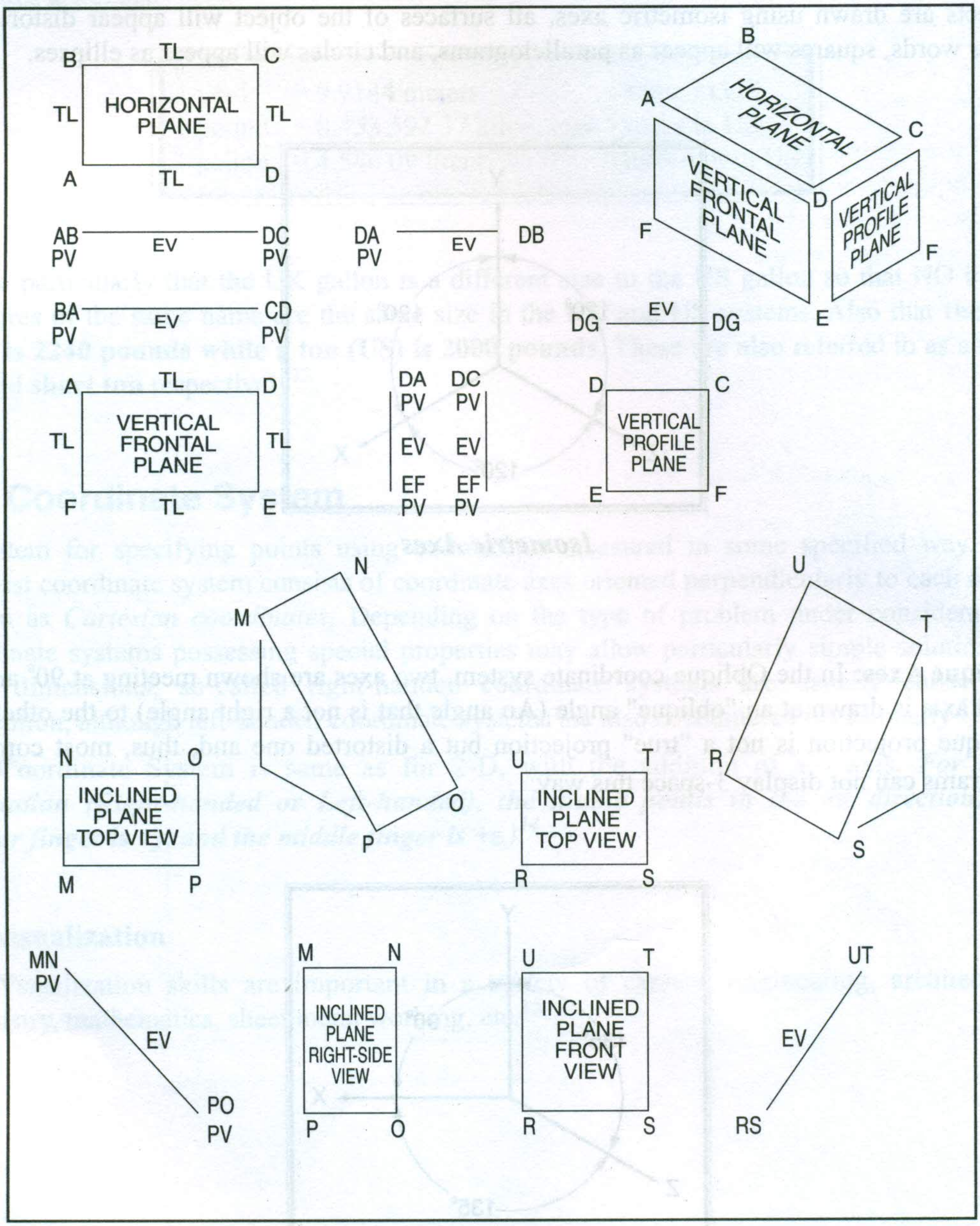
Oblique Axes: In the Oblique coordinate system, two axes are shown meeting at 90^0 and the third axis is drawn at an "oblique" angle (An angle that is not a right angle) to the other two. Oblique projection is not a "true" projection but a distorted one and, thus, most computer programs can not display 3-space this way.



Oblique Axes

Types of Planes

The Multiview and Isometric Drawings shown below illustrate four types of Planes: HORIZONTAL, FRONTAL, PROFILE and INCLINED [57].



TS - True Shape
PV - Point View
EV - Edge View
TL - True Length